(Please write your Exam Roll No.) Exam Roll No. ....

END TERM EXAMINATION

SECOND SEMESTER [B.TECH] MAY-JUNE 2026

Paper Code: BS-112 Subject: Applied Mathematics-II

Time: 3 Hours Maximum Marks: 60

Note: Attempt all questions as directed. Internal choice is indicated.

Q1 Attempt **any four** of the following questions: **(4x5=20)**

(a) Determine the analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$

(b) Evaluate $\int_0^{3+i} z^2\,dz$ along the parabola $y^2 = \frac{x}{3}$.

(c) State and prove Second shifting property of Laplace Transformation.

(d) Show that the function $f(z) = z|z|$ is not analytic anywhere.

(e) Find the linear fractional transformation (Mobius transformation) which maps the points $z = 0, 1, \infty$ to the points $w = -1, -i$ and 1 respectively.

(f) Obtain Laplace transform of $f(t) = t^2 e^t \sin t$.

(g) Find the Fourier cosine transform of $f(x) = e^{-x} + e^{-2x}$, $x > 0$

(h) Form a Partial Differential Equation from $z = f\left(\frac{y}{x}\right) + g(xy)$.

Q2 (a) Determine the poles of the function $f(z) = \frac{z^2}{(z-1)(z-2)^2}$ and the residue at each pole. **(5)**

(b) Prove that the function $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$, $z \neq 0$ and $f(0) = 0$ is continuous and Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$does not exist. **(5)**

OR

Q3 (a) Evaluate $\int_C \frac{dz}{(z-1)(z-2)(z-3)}\,dz$ where C is the circle $|z| = 4$ **(5)**

(b) Find the Laurent series of $f(z) = \frac{7z-2}{z^3-z^2-2z}$ in the region (i) $0 < |z+1| < 1$ (ii) $1 < |z+1| < 3$ **(5)**

Q4 (a) Obtain the condition under which the mapping $w = \frac{az+b}{cz+d}$ maps a straight line of z-plane into a unit circle of w-plane. **(5)**

(b) Using residue theorem, evaluate $\int_C \frac{e^z\,dz}{(z+1)^2(z-2)}$ where $C: |z-1| = 3$ **(5)**

OR

Q5 (a) An electrostatic field in the xy-plane is given by the potential function $\phi = 3x^2y - y^3$. Find the Stream function. **(5)**

(b) Evaluate $\int_0^{\pi} \frac{a\,d\theta}{a^2+\sin^2\theta}$ $(0 < a < 1)$ **(5)**

Q6 (a) Find the Laplace transform of (i) $\left[\frac{\cos at - \cos bt}{t}\right]$, (ii) $\frac{s+2}{(s^2-4s+13)}$ **(5)**

(b) Solve, using Laplace Transform technique, the differential equation **(5)**

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t \quad \text{where } y(0) = 0,\ y'(0) = 1$$

P.T.O.

OR

Q7 (a) Obtain the Fourier series for the function $f(x)$ given by (5)

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} \, for \; -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi} \, for \quad 0 \le x \le \pi \end{cases}$$

(b) Using Fourier transform to solve the equation $\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial x^2}, x > 0, t > 0$ subject to the conditions (i) $u(0,t) = 0, t > 0$ (ii) $u(x,0) = \begin{cases} 1, 0 < x < 1 \\ 0, \quad x \ge 1 \end{cases}$ (iii) $u(x,t)$ is bounded. (5)

Q8 (a) Using the method of separation of variables, solve the equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0)6e^{-3x}$ (5)

(b) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = a \sin\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, show that the displacement of any point at a distance x from one end at time t is given by (5)

$y(x,t) = a \sin\left(\frac{\pi x}{l}\right) \cos\left(\frac{\pi at}{l}\right)$.

OR

Q9 (a) A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0^0C and are kept at that temperature. Find the temperature function $u(x,t)$. (5)

(b) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary condition the rectangle $u(x,0) = 3 \sin n\pi x$, $u(0,t) = 0$, and $u(l,t) = 0$ where $0 < x < 1$, $t > 0$ (5)

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